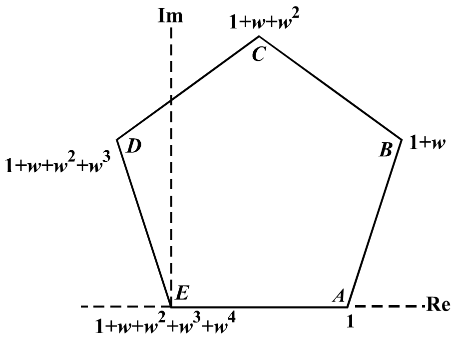
1	Direction of $l_1 = k[7, 0, -10]$ Direction of $l_2 = k[1, 3, -1]$	B1	For both directions
	EITHER $\mathbf{n} = [7, 0, -10] \times [1, 3, -1]$ *OR* $\begin{cases} [x, y, z] . [7, 0, -10] = 0 \Rightarrow 7x - 10z = 0 \\ [x, y, z] . [1, 3, -1] = 0 \Rightarrow x + 3y - z = 0 \end{cases}$	M1	For finding vector product of directions of l_1 and l_2 *OR* for using 2 scalar products and obtaining equations
	$\Rightarrow \mathbf{n} = k[10, -1, 7]$	A1	For correct **n**
	METHOD 1 Vector $(\mathbf{a} - \mathbf{b})$ from l_1 to $l_2 = \pm[4, 6, -10]$ *OR* $\pm[-4, 3, 1]$ *OR* $\pm[3, 3, -9]$ *OR* $\pm[-3, 6, 0]$	B1	For a correct vector
	$d = \dfrac{\lvert(\mathbf{a} - \mathbf{b}) . \mathbf{n}\rvert}{\lvert\mathbf{n}\rvert} = \dfrac{36}{\sqrt{150}}$	M1*	For finding $(\mathbf{a} - \mathbf{b}) . \mathbf{n}$
		M1 (*dep)	For $\lvert\mathbf{n}\rvert$ in denominator *OR* for using $\hat{\mathbf{n}}$
	$d = \frac{6}{5}\sqrt{6} \approx 2.94$	A1 **7**	For correct distance **AEF**
	METHOD 2 Planes containing l_1 and l_2 perp. to **n**	M1*	For finding planes and $p_1 - p_2$ seen
	are $\mathbf{r} . [10, -1, 7] = p_1 = 70$, $\mathbf{r} . [10, -1, 7] = p_2 = 34$	B1	For $p_1 = 70k$ and $p_2 = 34k$
	$\Rightarrow d = \dfrac{\lvert 70 - 34\rvert}{\sqrt{150}} = \dfrac{36}{\sqrt{150}} = \frac{6}{5}\sqrt{6} \approx 2.94$	M1 (*dep)	For $\lvert\mathbf{n}\rvert$ in denominator *OR* for using $\hat{\mathbf{n}}$
		A1	For correct distance **AEF**
	METHOD 3 $\mathbf{r}_1 = [7\lambda, 0, 10 - 10\lambda]$ *OR* $[7 + 7\lambda, 0, -10\lambda]$ $\mathbf{r}_2 = [4 + \mu, 6 + 3\mu, -\mu]$ *OR* $[3 + \mu, 3 + 3\mu, 1 - \mu]$	B1	For correct points on l_1 and l_2 using different parameters
	$\begin{array}{rcl|c|c|c} 7\lambda + 10\alpha - \mu & = & 4 & -3 & 3 & -4 \\ -\alpha - 3\mu & = & 6 & 6 & 3 & 3 \\ -10\lambda + 7\alpha + \mu & = & -10 & 0 & -9 & 1 \end{array}$ $\Rightarrow \alpha = -\frac{6}{25}$	M1*	For setting up 3 linear equations from $\mathbf{r}_1 + \alpha\mathbf{n} = \mathbf{r}_2$ and solving for α
	$\lvert\mathbf{n}\rvert = \sqrt{150}$	M1 (*dep)	For $\lvert\mathbf{n}\rvert$ seen multiplying α
	$\Rightarrow d = \frac{6}{25}\sqrt{150} = \frac{6}{5}\sqrt{6} \approx 2.94$	A1	For correct distance **AEF**
		$\boxed{7}$	

2	**(i)**	$ar = r^5a \Rightarrow rar = r^6a$	M1		Pre-multiply $ar = r^5a$ by r
		$r^6 = e \Rightarrow rar = a$	A1	**2**	Use $r^6 = e$ and obtain answer **AG**
	(ii)	METHOD 1			
		For n = 1, $rar = a$ *OR* For n = 0, $r^0 ar^0 = a$	B1		For stating true for n = 1 *OR* for n = 0
		Assume $r^k ar^k = a$			
		EITHER Assumption $\Rightarrow$ $r^{k+1}ar^{k+1} = rar = a$	M1		For attempt to prove true for k + 1
		OR $r^{k+1}ar^{k+1} = r.r^k ar^k.r = rar = a$			
		OR $r^{k+1}ar^{k+1} = r^k.rar.r^k = r^k ar^k = a$	A1		For obtaining correct form
		Hence true for all $n \in \mathbb{Z}^+$	A1	**4**	For statement of induction conclusion
		METHOD 2			
		$r^2ar^2 = r.rar.r = rar = a$, similarly for $r^3ar^3 = a$	M1		For attempt to prove for n = 2, 3
		$r^4ar^4 = r.r^3ar^3.r = rar = a$, similarly for $r^5ar^5 = a$	A1		For proving true for n = 2, 3, 4, 5
		$r^6ar^6 = eae = a$	B1		For showing true for n = 6
		For n > 6, $r^n = r^{n \bmod 6}$, hence true for all $n \in \mathbb{Z}^+$	A1		For using n mod 6 and correct conclusion
		METHOD 3			
		$r^n ar^n = r^{n-1}.rar.r^{n-1}$ *OR* $r^n ar^n = r^n.r^5a.r^{n-1} = r^{n+5}ar^{n-1}$	M1		Starting from n, for attempt to prove true for $n-1$
		$= r^{n-1}ar^{n-1}$	A1		For proving true for $n-1$
		$= r^{n-2}ar^{n-2} = \dots$	A1		For continuation from $n-2$ downwards
		$= rar = a$	B1		For final use of $rar = a$ **SR** can be done in reverse
		METHOD 4			
		$ar = r^5a \Rightarrow ar^2 = r^5ar = r^{10}a$ etc.	M1		For attempt to derive $ar^n = r^{5n}a$
		$\Rightarrow ar^n = r^{5n}a$	A1		For correct equation **SR** may be stated without proof
		$\Rightarrow r^n ar^n = r^{6n}a$	B1		For pre-multiplication by r^n
		$= ea = a$	A1		For obtaining a ($r^6 = e$ may be implied)
			$\boxed{6}$		

3					Allow $\text{cis}\frac{k}{5}\pi$ and $e^{\frac{k}{5}\pi i}$ throughout
	(i)	$w^2 = \cos\frac{4}{5}\pi + i\sin\frac{4}{5}\pi$	B1		For correct value
		$w^3 = \cos\frac{6}{5}\pi + i\sin\frac{6}{5}\pi$	B1		For correct value
		$w^* = \cos\frac{2}{5}\pi - i\sin\frac{2}{5}\pi$	B1		For w^* seen or implied
		$= \cos\frac{8}{5}\pi + i\sin\frac{8}{5}\pi$	B1	**4**	For correct value **SR** For exponential form with i missing, award B0 first time, allow others
	(ii)		B1*		For $1+w$ in approximately correct position
			B1 (*dep)		For $AB \approx BC \approx CD$
			B1 (*dep)		For BC, CD equally inclined to Im axis
			B1	**4**	For E at the origin Allow points joined by arcs, or not joined Labels not essential
	(iii)	$z^5 - 1 = 0$ *OR* $z^5 + z^4 + z^3 + z^2 + z = 0$	B1	**1**	For correct equation **AEF (**in any variable) Allow factorised forms using w, exp or trig
				$\boxed{9}$	
4	**(i)**	$y = xz \Rightarrow \frac{dy}{dx} = z + x\frac{dz}{dx}$	B1		For correct differentiation of substitution
		$\Rightarrow xz + x^2\frac{dz}{dx} - xz = x\cos z \Rightarrow x\frac{dz}{dx} = \cos z$	M1 A1		For substituting into DE For DE in variables separable form
		$\Rightarrow \int \sec z \, dz = \int \frac{1}{x} dx$	M1		For attempt at integration to ln form on LHS
		$\Rightarrow \ln(\sec z + \tan z) = \ln kx$ *OR* $\ln\tan\left(\frac{1}{2}z + \frac{1}{4}\pi\right) = \ln kx$	A1		For correct integration (k not required here)
		$\Rightarrow \sec\left(\frac{y}{x}\right) + \tan\left(\frac{y}{x}\right) = kx$ *OR* $\tan\left(\frac{y}{2x} + \frac{1}{4}\pi\right) = kx$	A1	**6**	For correct solution **AEF** including $\text{RHS} = e^{(\ln x)+c}$
	(ii)	$(4, \pi) \Rightarrow \sec\frac{1}{4}\pi + \tan\frac{1}{4}\pi = 4k$ *OR* $\tan\left(\frac{1}{8}\pi + \frac{1}{4}\pi\right) = 4k$	M1		For substituting $(4, \pi)$ into their solution (with k)
		$\Rightarrow \sec\left(\frac{y}{x}\right) + \tan\left(\frac{y}{x}\right) = \frac{1}{4}\left(1+\sqrt{2}\right)x$ *OR* $\tan\left(\frac{y}{2x} + \frac{1}{4}\pi\right) = \left(\frac{1}{4}\tan\frac{3}{8}\pi\right)x$ *or* $\frac{1}{4}\left(1+\sqrt{2}\right)x$	A1	**2**	For correct solution **AEF** Allow decimal equivalent $0.60355x$ Allow $e^{\ln x}$ for x
				$\boxed{8}$	

5	**(i)**	$C + \mathrm{i}\,S = 1 + \frac{1}{2}\mathrm{e}^{\mathrm{i}\theta} + \frac{1}{4}\mathrm{e}^{2\mathrm{i}\theta} + \frac{1}{8}\mathrm{e}^{3\mathrm{i}\theta} + \dots$	M1 A1		For using $\cos n\theta + \mathrm{i}\sin n\theta = \mathrm{e}^{\mathrm{i}\,n\theta}$ at least once for $n \geqslant 2$ For correct series
		$= \dfrac{1}{1-\frac{1}{2}\mathrm{e}^{\mathrm{i}\theta}} = \dfrac{2}{2-\mathrm{e}^{\mathrm{i}\theta}}$	M1 A1	**4**	For using sum of infinite GP For correct expression **AG** **SR** For omission of 1st stage award up to M0 A0 M1 A1 **OEW**
	(ii)	$C + \mathrm{i}\,S = \dfrac{2\left(2-\mathrm{e}^{-\mathrm{i}\theta}\right)}{\left(2-\mathrm{e}^{\mathrm{i}\theta}\right)\left(2-\mathrm{e}^{-\mathrm{i}\theta}\right)}$	M1		For multiplying top and bottom by complex conjugate
		$= \dfrac{4-2\mathrm{e}^{-\mathrm{i}\theta}}{4-2\left(\mathrm{e}^{\mathrm{i}\theta}+\mathrm{e}^{-\mathrm{i}\theta}\right)+1} = \dfrac{4-2\cos\theta+2\mathrm{i}\sin\theta}{4-4\cos\theta+1}$	M1		For reverting to cosθ and sinθ and equating Re *OR* Im parts
		$\Rightarrow C = \dfrac{4-2\cos\theta}{5-4\cos\theta},\quad S = \dfrac{2\sin\theta}{5-4\cos\theta}$	A1 A1	**4**	For correct expression for C **AG** For correct expression for S
				$\boxed{8}$	
6	**(i)**	Aux. equation $m^2 + 2m + 17 (= 0)$ $\Rightarrow m = -1 \pm 4\mathrm{i}$	M1 A1		For attempting to solve correct auxiliary equation For correct roots
		CF $(y =)\ \mathrm{e}^{-x}\left(A\cos 4x + B\sin 4x\right)$	A1√		For correct CF (allow $A\begin{smallmatrix}\cos\\\sin\end{smallmatrix}(4x+\varepsilon)$) (trig terms required, not $\mathrm{e}^{\pm 4\mathrm{i}\,x}$) f.t. from their m with 2 arbitrary constants
		PI $(y =)\ px + q \Rightarrow\ 2p + 17(px+q) = 17x + 36$	M1		For stating and substituting PI of correct form
		$\Rightarrow\ p = 1$	A1		For correct value of p
		and $q = 2$	A1		For correct value of q
		GS $y = \mathrm{e}^{-x}\left(A\cos 4x + B\sin 4x\right) + x + 2$	B1√	**7**	For GS. f.t. from their CF+PI with 2 arbitrary constants in CF and none in PI. Requires $\boxed{y =}$.
	(ii)	$x \gg 0 \Rightarrow \mathrm{e}^{-x} \to 0$ *OR* very small $\Rightarrow y = x + 2$ approximately	B1 B1√	**2**	For correct statement. Allow graph For correct equation Allow ≈ , → and in words Allow relevant f.t. from linear part of GS
				$\boxed{9}$	

7	(i)	$(1, 3, 5)$ and $(5, 2, 5) \Rightarrow \pm[4, -1, 0]$ in Π	M1		For finding a vector in Π
		$\mathbf{n} = [2, -2, 3] \times [4, -1, 0] = k[1, 4, 2]$	M1		For finding vector product of direction vectors of l and a line in Π
			A1		For correct **n**
		$\Rightarrow \mathbf{r}.[1, 4, 2] = 23$	A1	**4**	For correct equation. Allow multiples
	(ii)	METHOD 1			
		Perpendicular to Π through $(-7, -3, 0)$ meets Π	M1		For using perpendicular from point on l to Π Award mark for $k\mathbf{n}$ used
		where $(-7+k)+4(-3+4k)+2(2k)=23$	M1		For substituting parametric line coords into Π
		$\Rightarrow k=2 \quad \Rightarrow d = 2\sqrt{1^2+4^2+2^2} = 2\sqrt{21} \approx 9.165$	M1		For normalising the **n** used in this part
			A1	**4**	For correct distance **AEF**
		METHOD 2			
		Π is $x+4y+2z=23$	M1		For attempt to use formula for perpendicular distance
		$\Rightarrow d = \dfrac{\lvert(-7)+4(-3)+2(0)-23\rvert}{\sqrt{1^2+4^2+2^2}} = 2\sqrt{21} \approx 9.165$	M1		For substituting a point on l into plane equation
			M1		For normalising the **n** used in this part
			A1		For correct distance **AEF**
		METHOD 3			
		$\mathbf{m} = [1, 3, 5]-[-7, -3, 0] = (\pm)[8, 6, 5]$	M1		For finding a vector from l to Π
		OR $= [5, 2, 5]-[-7, -3, 0] = (\pm)[12, 5, 5]$			
		$\Rightarrow d = \dfrac{\mathbf{m}.[1, 4, 2]}{\sqrt{1^2+4^2+2^2}} = \dfrac{42}{\sqrt{21}} = 2\sqrt{21} \approx 9.165$	M1		For finding $\mathbf{m}.\mathbf{n}$
			M1		For normalising the **n** used in this part
			A1		For correct distance **AEF**
		METHOD 4			As Method 1, using parametric form of Π
		$[-7, -3, 0]+k[1, 4, 2] = [1, 3, 5]+s[2, -2, 3]+t[4, -1, 0]$	M1		For using perpendicular from point on l to Π Award mark for $k\mathbf{n}$ used
		$\left.\begin{aligned} k-2s-4t &= 8 \\ 4k+2s+t &= 6 \\ 2k-3s &= 5 \end{aligned}\right\} \Rightarrow k=2 \quad \left(s=-\tfrac{1}{3},\ t=-\tfrac{4}{3}\right)$	M1		For setting up and solving 3 equations
		$\Rightarrow d = 2\sqrt{1^2+4^2+2^2} = 2\sqrt{21} \approx 9.165$	M1		For normalising the **n** used in this part
			A1		For correct distance **AEF**
		METHOD 5			
		$d_1 = \dfrac{23}{\sqrt{1^2+4^2+2^2}} = \dfrac{23}{\sqrt{21}}$	M1		For attempt to find distance from O to Π *OR* from O to parallel plane containing l
		$d_2 = \dfrac{[-7, -3, 0].[1, 4, 2]}{\sqrt{1^2+4^2+2^2}} = \dfrac{-19}{\sqrt{21}}$	M1		For normalising the **n** used in this part
		$\Rightarrow d_1-d_2 = d = \dfrac{23-(-19)}{\sqrt{21}} = 2\sqrt{21} \approx 9.165$	M1		For finding d_1-d_2
			A1		For correct distance **AEF**
	(iii)	$(-7, -3, 0)+k(1, 4, 2)$	M1		State or imply coordinates of a point on the reflected line
		Use $k=4$	M1		State or imply 2 × distance from **(ii)** Allow $k=\pm4$ *OR* $\pm4\sqrt{21}$ f.t. from **(ii)**
		$\mathbf{b} = [2, -2, 3]$	B1		For stating correct direction
		$\mathbf{a} = [-3, 13, 8]$	A1	**4**	For correct point seen in equation $\mathbf{r} = \mathbf{a}+t\mathbf{b}$
		$\mathbf{r} = [-3, 13, 8]+t[2, -2, 3]$			**AEF** in this form
				12	

8 (i)	$\{A, D\}$ *OR* $\{A, E\}$ *OR* $\{A, F\}$	B1	**1**	For stating any one subgroup
(ii)	*A* is the identity 5 is not a factor of 6 *OR* elements can be only of order 1, 2, 3, 6	B1 B1	 **2**	For identifying *A* as the identity For reference to factors of 6
(iii)	$BE = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = D$, $EB = \begin{pmatrix} 0 & \omega \\ \omega^2 & 0 \end{pmatrix} = F$	M1 A1 A1		For finding *BE* and *EB* AND using $\omega^3 = 1$ For correct *BE* (*D or* matrix) For correct *EB* (*F or* matrix)
	D *or* $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, F *or* $\begin{pmatrix} 0 & \omega \\ \omega^2 & 0 \end{pmatrix} \in M$ $\Rightarrow$ closure property satisfied	A1	**4**	For justifying closure
(iv)	$B^{-1} = \frac{1}{1}\begin{pmatrix} \omega^2 & 0 \\ 0 & \omega \end{pmatrix} = C$	M1 A1		For correct method of finding either inverse For correct $B^{-1} = C$ Allow $\begin{pmatrix} \omega^2 & 0 \\ 0 & \omega \end{pmatrix}$
	$E^{-1} = \frac{1}{-1}\begin{pmatrix} 0 & -\omega^2 \\ -\omega & 0 \end{pmatrix} = E$	A1	**3**	For correct $E^{-1} = E$ Allow $\begin{pmatrix} 0 & \omega^2 \\ \omega & 0 \end{pmatrix}$
(v)	METHOD 1 *M* is not commutative e.g. from $BE \neq EB$ in part **(iii)** *N* is commutative (as × mod 9 is commutative) $\Rightarrow$ *M* and *N* not isomorphic	B1 B1 B1#	 **3**	For justification of *M* being not commutative For statement that *N* is commutative For correct conclusion
	METHOD 2 Elements of *M* have orders 1, 3, 3, 2, 2, 2 Elements of *N* have orders 1, 6, 3, 2, 3, 6 Different orders *OR* self-inverse elements $\Rightarrow$ *M* and *N* not isomorphic	B1* B1 (*dep) B1#		For all orders of one group correct For sufficient orders of the other group correct For correct conclusion **SR** Award up to B1 B1 B1 if the self-inverse elements are sufficiently well identified for the groups to be non-isomorphic
	METHOD 3 *M* has no generator since there is no element of order 6 *N* has 2 *OR* 5 as a generator $\Rightarrow$ *M* and *N* not isomorphic	B1 B1 B1#		For all orders of *M* shown correctly For stating that *N* has generator 2 *OR* 5 For correct conclusion
	METHOD 4 (tables below)	B1* B1 (*dep)		For stating correctly all 6 squared elements of one group For stating correctly sufficient squared elements of the other group
	$\Rightarrow$ *M* and *N* not isomorphic	B1#		For correct conclusion
				# In all Methods, the last B1 is dependent on at least one preceding B1
			13	

METHOD 4 tables:

M	*A*	*B*	*C*	*D*	*E*	*F*
A	*A*	*B*	*C*	*D*	*E*	*F*
B	*B*	*C*	*A*	*F*	*D*	*E*
C	*C*	*A*	*B*	*E*	*F*	*D*
D	*D*	*E*	*F*	*A*	*B*	*C*
E	*E*	*F*	*D*	*C*	*A*	*B*
F	*F*	*D*	*E*	*B*	*C*	*A*

N	1	2	4	8	7	5
1	1	2	4	8	7	5
2	2	4	8	7	5	1
4	4	8	7	5	1	2
8	8	7	5	1	2	4
7	7	5	1	2	4	8
5	5	1	2	4	8	7